## GCE Examinations

## Advanced Subsidiary / Advanced Level

## Decision Mathematics

Module D1

## Paper C

## MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.
Accuracy marks (A) can only be awarded when a correct method has been used.
(B) marks are independent of method marks.


Written by Shaun Armstrong \& Dave Hayes
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## D1 Paper C - Marking Guide

1. (a)

(b) e.g. $A B C D E A$ is a Hamiltonian cycle choose $A C$ inside so $B D$ and $B C$ must go outside put $A D$ or $C E$ inside, then the other cannot be placed without overlapping so no planar drawing is possible
(c)

e.g. $A E B F C D A$ is a Hamiltonian cycle, redraw as polygon:

choose $A D$ inside so $B F$ and $C E$ must go outside but this creates a crossing outside so no planar drawing is possible
2. e.g.

3. (a)

| 150 | 104 | 20060 | 184 | 84 | 120 | (pivot in box) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 150 | 104 | 200184 | 84 | 120 | 60 |  |
| $L_{1}$ |  |  |  |  |  |  |
| $\underbrace{200}$ | 184 | $\underbrace{150 \quad 104}$ | 84 | 120 | 60 |  |
| $L_{2}$ | $L_{3}$ |  |  |  |  |  |
| 200 | 184 | $\underbrace{150 \quad 104}$ | 120 | 84 | 60 |  |
|  |  | $L_{4}$ |  |  |  |  |
| 200 | 184 | $\underbrace{150 \quad 120}$ | 104 | 84 | 60 |  |
| $L_{5}$ |  |  |  |  |  |  |
| 200 | 184 | $\underbrace{150} 120$ | 104 | 84 | 60 |  |
|  |  | $L_{6}$ |  |  |  | now complete |

M2 A2
(b) sort list in decreasing order and have bins of size 240
take each length in turn and put it in the first bin in which it can fit count number of bins used

$\therefore 5$ bins needed
(c) unused rod $=(5 \times 240)-(200+184+150+120+104+84+60)$

$$
=298 \therefore \text { not possible }
$$

B1 (9)

label $H$ - label $Y=37=$ weight $Y H$
label $Y$ - label $L e=23=$ weight $L e Y$
label $L e$ - label $M a=40=$ weight MaLe
label $M a$ - label $L i=35=$ weight $L i M a \quad$ M1 A1
so Li Ma Le Y H is shortest route, length $=135$ miles
A2
(10)
5. (a) arcs in ascending order by inspection:
$20,25,25,35,38,42,50,52,55,68,75,85,85,93,100,105,108,175$

order: $\mathrm{E}-\mathrm{Gr}, \mathrm{Gr}-\mathrm{F}, \leftrightarrow \mathrm{I}-\mathrm{Ge}, \mathrm{Gr}-\mathrm{C}, \mathrm{Gr}-\mathrm{I}, \mathrm{Gr}-\mathrm{R}, \mathrm{C}-\mathrm{U}$;
M2 A1
cost $£ 263$
(b) (i) $25,50,55,68,75,85,85,93,100,105,108,175$


$$
\mathrm{I}-\mathrm{Ge}, \mathrm{I}-\mathrm{F}, \mathrm{C}-\mathrm{U}, \mathrm{C}-\mathrm{E}, \mathrm{~F}-\mathrm{R}(\text { or } \mathrm{Ge}-\mathrm{R}), \mathrm{Ge}-\mathrm{E} \text {; cost } £ 396
$$

(ii) previous tree still minimum, cost $=£ 263$
(c) e.g. translations between other languages cheaper via Greek even though Greek translation not required
(d) an asymmetric array could show both costs
(e) Prim's B1
(f) e.g. that a translation via another language will be of as good quality as one done directly - unlikely to be the case

B2
6. (a)

(b) initial matching shown by $工$
search for alternating path giving e.g. $G-S$ (breakthrough)
change status giving $G=S$
alternating path e.g. $E-A=H-O=F-D=I-C$ (breakthrough)
change status giving $E=A-H=O-F=D-I=C$
complete matching e.g. $E-A, F-D, G-S, H-O, I-C$
(c) e.g. there is now a cycle: $H-C=I-D=F-O=H$
change status giving $H=C-I=D-F=O-H$
alternative matching $E-A, F-O, G-S, H-C, I-D$
M2 A1
7. (a) $6 x+15 y+12 z \leq 185$
$3 x+3 y+z \leq 30$
$x+4 y+4 z \leq 60$
(b) there are 3 independent variables
(c) rewriting with slack variables gives
$6 x+15 y+12 z+r=185$
$3 x+3 y+z+s=30$
$x+4 y+4 z+t=60$
need to maximise $I=40 x+90 y+60 z$, considering 10's of pounds gives
objective function $P-4 x-9 y-6 z=0$, hence given tableau
(d) $\theta$ values are $12 \frac{1}{3}, 10$ and 15 so pivot row is $2^{\text {nd }}$ row

| Basic Var. | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r$ | -9 | 0 | 7 | 1 | -5 | 0 | 35 |
| $y$ | 1 | 1 | $\frac{1}{3}$ | 0 | $\frac{1}{3}$ | 0 | 10 |
| $t$ | -3 | 0 | $\frac{8}{3}$ | 0 | $-\frac{4}{3}$ | 1 | 20 |
| $P$ | 5 | 0 | -3 | 0 | 3 | 0 | 90 |

increase $z$ next, $\theta$ values are 5,30 and $7 \frac{1}{2}$ so pivot row is 1 st row

| Basic Var. | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $z$ | $-\frac{9}{7}$ | 0 | 1 | $\frac{1}{7}$ | $-\frac{5}{7}$ | 0 | 5 |
| $y$ | $\frac{10}{7}$ | 1 | 0 | $-\frac{1}{21}$ | $\frac{4}{7}$ | 0 | $8 \frac{1}{3}$ |
| $t$ | $\frac{3}{7}$ | 0 | 0 | $-\frac{8}{21}$ | $\frac{4}{7}$ | 1 | $6 \frac{2}{3}$ |
| $P$ | $\frac{8}{7}$ | 0 | 0 | $\frac{3}{7}$ | $\frac{6}{7}$ | 0 | 105 |

optimal solution as all values on the objective row are $\geq 0$
(e) 0 of $X, 8 \frac{1}{3}$ of $Y$ and 5 of $Z$, giving $P=105$ so profit $=£ 1050$
(f) try integer coordinates around the optimal solution
e.g. $(0,8,5)(1,8,5)(0,9,5)$ etc. checking feasible and seeking optimum B2

## Performance Record - D1 Paper C

| Question no. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Topic(s) | $\begin{aligned} & \hline \begin{array}{l} \text { graphs, } \\ \text { planarity } \end{array} \end{aligned}$ | activity network | $\begin{aligned} & \hline \text { quick sort, } \\ & \text { bin } \\ & \text { packing } \end{aligned}$ | Dijkstra's | Kruskal's | matching | simplex |  |
| Marks | 6 | 7 | 9 | 10 | 12 | 13 | 18 | 75 |
| Student |  |  |  |  |  |  |  |  |
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