Chapter 4

Exercise 4A

1. **(a)** 2×2
(b) 2×1
(c) 2×3
(d) 1×3
(e) 1×2
(f) 3×3

2. **(a)** $\begin{pmatrix} 8 & -1 \\ 1 & 4 \end{pmatrix}$
(b) $\begin{pmatrix} 2 & 2 \\ -2 & 5 \end{pmatrix}$
(c) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

3. **(a)** Not possible
(b) $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 1 & 4 \end{pmatrix}$
(d) Not possible
(e) $\begin{pmatrix} 3 & -1 & 4 \end{pmatrix}$
(f) Not possible
(g) $\begin{pmatrix} -3 & 1 & -4 \end{pmatrix}$

4. $a = 6, b = 3, c = 2, d = -1$

5. $a = 4, b = 3, c = 5$

6. $a = 2, b = -2, c = 2, d = 1, e = -1, f = 3$

Exercise 4B

1. **(a)** $\begin{pmatrix} 6 & 0 \\ 12 & -18 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 0 \\ 2 & -3 \end{pmatrix}$
(c) $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$

2. $k = 3, x = -1$

3. $a = 3, b = -3.5, c = -1, d = 2$

4. $a = 5, b = 5, c = -2, d = 2$

5. $k = \dfrac{3}{2}$

Exercise 4C

1. **(a)** 1×2
(b) 3×3
(c) 1×2
(d) 2×2
(e) 2×3
(f) 3×2

2. **(a)** $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$
(b) $\begin{pmatrix} -2 & 1 \\ -4 & 7 \end{pmatrix}$

3. **(a)** $\begin{pmatrix} -3 & -2 & -1 \\ 3 & 3 & 0 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & -4 \\ 0 & 9 \end{pmatrix}$

4. **(a)** Not possible
(b) $\begin{pmatrix} -6 & -4 \\ -3 & -2 \end{pmatrix}$
(c) Not possible
(d) $\begin{pmatrix} 7 \\ 0 \end{pmatrix}$
(e) (-8)
(f) $\begin{pmatrix} -7 & -7 \end{pmatrix}$

5. $\begin{pmatrix} 2 & 6-a & 2a \\ 1 & 4 & -2 \end{pmatrix}$

6. $\begin{pmatrix} 3x+2 & 0 \\ 0 & 3x+2 \end{pmatrix}$

7. **(a)** $\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 2 \times k \\ 0 & 1 \end{pmatrix}$

8. **(a)** $\begin{pmatrix} a^2 & 0 \\ ab & 0 \end{pmatrix}$
(b) 3

9. (a) $\begin{pmatrix} -8 & -14 \\ -4 & -7 \\ 0 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} -16 & 29 \end{pmatrix}$

10. (a) $\begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$

(b) $\begin{pmatrix} -3 & 2 & 3 \end{pmatrix}$

Exercise 4D

1. (a) Not linear
(b) Not linear
(c) Not linear
(d) Linear
(e) Not linear
(f) Linear

2. (a) Linear: $\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$
(b) Not linear
(c) Not linear
(d) Linear: $\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$
(e) Linear: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

3. (a) Not linear
(b) Linear: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
(c) Linear: $\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$
(d) Linear: $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$
(e) Linear: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

4. (a) $\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$
(b) $\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$

5. (a) (1, 1), (−2, 3), (−5, 1)
(b) (3, −2), (14, 6), (9, −2)
(c) (−2, −2), (−6, 4), (−2, 10)

6. (a) (−2, 0), (0, 3), (2, 0), (0, −3)
(b) (−1, −1), (−1, 1), (1, 1), (1, −1)
(c) (−1, −1), (1, −1), (1, 1), (−1, 1)

Exercise 4E

1. (a) Reflection in x-axis (or line $y = 0$)
(b) Rotation 90° anticlockwise about (0, 0)
(c) Rotation 90° clockwise about (0, 0)

2. (a) Enlargement scale factor ½, centre (0, 0)
(b) Reflection in line $y = x$
(c) No change (this is the Identity matrix)

3. (a) Rotation 45° clockwise about (0, 0)
(b) Enlargement scale factor 4, centre (0, 0)
(c) Rotation 225° anticlockwise about (0, 0)

4. (a) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
(c) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

5. (a) $\begin{pmatrix} -4 & 0 \\ 0 & -4 \end{pmatrix}$
(b) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
(c) $\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

Exercise 4F

1. **(a)** $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; Reflection in $y = x$

(b) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; Reflection in $y = x$

(c) $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$; Enlargement scale factor −2, centre (0, 0)

(d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; Identity (no transformation)

(e) $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$; Enlargement scale factor 4, centre (0, 0)

2. **(a)** $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $D = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) **(i)** Reflection in y-axis
(ii) Reflection in y-axis
(iii) Rotation of 180° about (0, 0)
(iv) Reflection in $y = -x$
(v) No transformation (Identity)
(vi) Rotation of 90° anticlockwise about (0, 0)
(vii) No transformation (Identity)

3. Reflection in $y = x$

5. **(a)** $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(b) Rotation of 90° anticlockwise about (0, 0)

(c) Rotation of 45° anticlockwise about (0, 0)

(d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (Identity matrix)

6. **(a)** $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) Reflection in y-axis

7. $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$, Reflection in the line $y = -x$

8. $\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$, Enlargement scale factor 8

Exercise 4G

1. **(a)** Non-singular. Inverse = $\begin{pmatrix} 1 & 0.5 \\ 2 & 1.5 \end{pmatrix}$

(b) Singular

(c) Singular

(d) Non-singular. Inverse = $\begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}$

(e) Singular

(f) Non-singular. Inverse = $\begin{pmatrix} -0.2 & 0.3 \\ 0.6 & -0.4 \end{pmatrix}$

2. **(a)** −3

(b) −5

(c) $\frac{1}{4}$

3. **(a)** $\begin{pmatrix} -(2+a) & 1+a \\ 1+a & -a \end{pmatrix}$

(b) $\begin{pmatrix} -\frac{1}{a} & -\frac{3}{1} \\ \frac{1}{b} & \frac{2}{b} \end{pmatrix}$ (provided a ≠ 0, b ≠ 0)

4. **(b)** $\begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix}$

5. **(a)** $\mathbf{B} = \mathbf{A}^{-1}\mathbf{C}$

(b) $\begin{pmatrix} 1 & 4 \\ -1 & 2 \end{pmatrix}$

6. **(a)** $\mathbf{A} = \mathbf{C}^{-1}$

(b) $\begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix}$

7. $\begin{pmatrix} 2 & 4 & -3 \\ 0 & 1 & 2 \end{pmatrix}$

8. $\begin{pmatrix} 1 & 3 \\ -2 & 1 \\ 0 & -1 \end{pmatrix}$

9. **(a)** $\frac{1}{2ab}\begin{pmatrix} 2b & -b \\ -4a & 3a \end{pmatrix}$

(b) $\begin{pmatrix} -3 & 2 \\ -1 & 3/2 \end{pmatrix}$

10. (a) $\det(\mathbf{A}) = 0$, $\det(\mathbf{B}) = 0$

(b) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Exercise 4H

1. (a) Rotation of 90° anticlockwise about (0, 0)

(b) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(c) Rotation of -90° anticlockwise about (0, 0)

2. (a) (i) Rotation of 180° about (0, 0)

(iii) Rotation of 180° about (0, 0)

(b) (i) Reflection in $y = -x$

(iii) Reflection in $y = -x$

(c) $\det(\mathbf{S}) = 1$, $\det(\mathbf{T}) = -1$

3. (a) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$; reflection in $y = 0$

(b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$; reflection in $y = 0$

(c) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$; reflection in $x = 0$

(d) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$; reflection in $x = 0$

Exercise 4I

1. (a) (0, 0), (−1, 3), (7, 19), (8, 16)

(b) 40

2. (a) (1, 2), (6, 2), (3, −1)

(c) 3.75

3. (a) $(2, -1), (3a - 9, -3a), (-8, 4), (3 - 3a, 3 + 3a)$

(b) $-a - 3$

(c) 2

4. (a) 70

(b) 30

(c) 15

(d) 90

(e) 90

(f) 210

5. **(a)** $a^2 + 2a - 9$

(b) −5, −3, 1 or 3

Exercise 4J

1. (a) $x = 3$, $y = -5$

(b) $x = 0.5$, $y = 3$

2. (a) $x = 2$, $y = -3$

(b) $x = -1$, $y = 4$

Mixed exercise 4K

1. $P = (7, -15)$, $Q = (2, -2)$, $R = (-4, 12)$

2. $\begin{pmatrix} 1 & 4 & 3 \\ -1 & 1 & -2 \end{pmatrix}$

3. (a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(b) Reflection in $y = x$

(c) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (Identity matrix)

4. (a) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} -2 & 3 \\ -1 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (Identity matrix)

5. (a) $\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$; reflection in x-axis and enlargement s.f. 2, centre (0, 0)

(b) $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$; reflection in x-axis and enlargement s.f. ½, centre (0, 0)

6. (a) $2p^2 - p$

(b) -1 or $\frac{3}{2}$

7. (a) $\begin{pmatrix} \frac{3}{a} & -\frac{1}{a} \\ -\frac{2}{b} & \frac{1}{b} \end{pmatrix}$

(b) $\begin{pmatrix} -1 & 1 \\ 4 & -1 \end{pmatrix}$

8. (a) $\mathbf{X} = \mathbf{BAB}^{-1}$

(b) $\begin{pmatrix} 6 & 2 \\ -4 & -3 \end{pmatrix}$