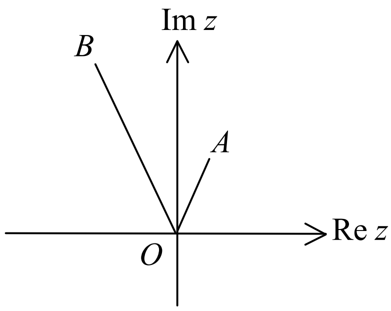
Further Pure Mathematics FP1 (6667)

Practice paper A mark scheme

Question number		Scheme	Marks
1.	(a)	$\mathbf{AB} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$	M1
		$= 10\mathbf{I}, \quad c = 10$	A1 (2)
	(b)	$\mathbf{A}^{-1} = \frac{1}{10}\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$	M1 A1 (2)
			(4 marks)
2.		$f(2) = 1\frac{1}{9} \qquad f(3) = -\frac{26}{27}$	B1
		$f(2.5) = 0.06415\ldots \qquad \Rightarrow 2.5 < \alpha < 3$	M1 A1
		$f(2.75) = -0.45125\ldots \qquad \Rightarrow 2.5 < \alpha < 2.75$	A1
			(4 marks)
3.	(a)	$f(k+1) = (2k+3)7^{k+1} - 1$	B1
		$f(k+1) - f(k) = (2k+3)7^{k+1} - 1 - [(2k+1)7^{k} - 1]$	
		$= (12k+20)7^{k} \qquad a = 12, b = 20$	M1 A1 (3)
	(b)	$f(1) = 3 \times 7 - 1 = 20$; divisible by 4	B1
		$f(k+1) - f(k) = 4\,(3k+5)7^{k}$	M1
		$\therefore$ true for $n = k + 1$ if true for $n = k$	A1
		Conclusion, with no wrong working seen.	A1 (4)
			(7 marks)
4.	(a)	$f'(x) = 3x^2 + 1$	M1 A1 (2)
	(b)	$f(1.2) = -0.072 \qquad f'(x) = 5.32$	B1
		$\alpha = 1.2 - \frac{-0.072}{5.32} = 1.21353\ldots = 1.21$ (3 sf)	M1 A1
		$\Rightarrow \alpha = 1.21$ (3 sig figs)	A1 cso (4)
			(6 marks)

Question number		Scheme	Marks
5.	(a)	Second root = 3 – i	B1
		Product of roots = (3 + i)(3 – i) = 10	
		or quadratic factor is $x^2 - 6x + 10$	M1 A1
		Complete method for third root or linear factor	M1
		Third root = $\frac{1}{2}$	A1 (5)
	(b)	Use candidate's 3 roots to find cubic with real coefficients	M1
		$(x^2 - 6x + 10)(2x - 1) = 2x^3 - 13x + 26x - 10$	
		Equating coefficients	M1
		$a = -13,\ b = 26$	A1 (3)
			(8 marks)
6.	(a)	$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$	B1 (1)
	(b)	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	M1 A1 (2)
	(c)	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}$	M1 A1 (2)
	(d)	$\begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} a+2 \\ b \end{pmatrix} = \begin{pmatrix} 3b \\ -3a-6 \end{pmatrix}$	M1
		$3b = 5a + 2,\ -3a - 6 = 2 - b$	M1
		Eliminate a or b	M1
		$a = -5.5,\ b = -8.5$	A1 A1 (5)
			(10 marks)
7.	(a)	$w = (2 + 2\text{i})(1 + \sqrt{3}\,\text{i})$	M1
		$= (2 - 2\sqrt{3}) + (2\sqrt{3} + 2)\,\text{i}$	A1, A1 (3)
	(b)	$\arg w = \arctan\left(\dfrac{2\sqrt{3}+2}{2-2\sqrt{3}}\right)$ or adds two	M1
		args e.g. $60^{\circ} + 45^{\circ}$	
		$= \dfrac{7\pi}{12}$ or 105° or 1.83 radians	A1 (2)

Question number	Scheme	Marks
(c)	$\lvert w \rvert = \sqrt{32} \quad (= 4\sqrt{2})$	M1 A1 (2)
(d)		B1 B1 (2)
	ft in quadrant other than first	
(e)	$\lvert AB \rvert^2 = 4 + 32 - 16\sqrt{2}\cos 45°$ (=20), then square root	M1
	AB = $2\sqrt{5}$	A1 (2)
(e)	Alternative: $w - z = 1 - 2\sqrt{3} + (2 + \sqrt{3})\mathrm{i}$	
	$\therefore AB = \lvert w - z \rvert = \sqrt{(1 - 2\sqrt{3})^2 + (2 + \sqrt{3})^2}$	M1
	$= \sqrt{20} = 2\sqrt{5}$	A1 (2)
		(11 marks)
8. (a)	$a = 3$	B1 (1)
(b)	$y = 2a^{\frac{1}{2}}x^{\frac{1}{2}} \quad \Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$	M1
	$y =$ and attempt $\frac{\mathrm{d}y}{\mathrm{d}x}$	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{t}$ sub $x = 3t^2$	M1
	Tangent is $y - 6t = \frac{1}{t}\left(x - 3t^2\right)$	M1
	$ty = x + 3t^2$ (✱)	A1 cso (4)
(c)	Equation of tangent at Q is $qy = x + 3q^2$	B1
	At R, $y = 0$ $\quad 0 = x + 3q^2$	M1
	$x = -3q^2$	A1 (3)

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	(d)	Equation of directrix is $x = -3$	B1
		$RD = 3q^2 - 3$	M1
		$3q^2 - 3 = 12 \quad \Rightarrow \quad q^2 = 5$	M1
		$q = \sqrt{5}$	A1 (4)
			(12 marks)
9.	(a)	If $n = 1$, $\sum_{r=1}^{n} r^2 = 1$, $\quad \frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}\times 1\times 2\times 3 = 1$	B1
		$\therefore$ true for $n = 1$	
		$\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$	M1 A1
		$= \frac{1}{6}(k+1)[k(2k+1) + 6(k+1)]$	M1
		$= \frac{1}{6}(k+1)(2k^2 + 7k + 6)$	
		$= \frac{1}{6}(k+1)(k+2)(2k+3)$	
		$= \frac{1}{6}(k+1)([k+1]+1)(2[k+1]+1)$	A1
		$\therefore$ true for $n = k + 1$ if true for $n = k$, $\therefore$ true for $n \in \mathbb{Z}^+$ by induction	A1 cso (6)
	(b)	Expand brackets and attempt to use appropriate formulae.	M1
		$\sum_{r=1}^{n} (r^2 + 6r + 5) = \frac{1}{6}n(n+1)(2n+1) + 6\times\frac{1}{2}n(n+1) + 5n$	M1 A1
		$= \frac{n}{6}[2n^2 + 3n + 1 + 18n + 18 + 30]$	A1
		$= \frac{n}{6}[2n^2 + 21n + 49]$	A1
		$= \frac{n}{6}(n+7)(2n+7) \qquad (✱)$	A1 cso (5)
	(c)	Use S(40) – S(9) $= \frac{40}{6}\times 47\times 87 - \frac{9}{6}\times 16\times 25$, = 26 660	M1, A1 (2)
			(13 marks)