Further Pure Mathematics FP1 (6667)

Practice paper B mark scheme

Question number		Scheme	Marks
1.	(a)	det **A** = $q(q-1)+6$	M1 A1 (2)
	(b)	$q^2-q+6=(q-\frac{1}{2})^2+5\frac{3}{4}$	M1 A1
		>0 for all real q $\Rightarrow$ **A** is non-singular (✱)	A1 cso (3)
	(b)	Alternative: If **A** is singular, det **A** = 0	
		$q^2-q+6=0$ 'b^2-4ac' $=1-24<0$	
		$\Rightarrow$ no real roots	M1 A1
		$\Rightarrow$ **A** is non-singular	A1 (3)
			(5 marks)
2.	(a)	$\left\|\frac{z}{w}\right\|=\sqrt{6^2+(-8)^2}$	M1
		$=10$	A1 (2)
	(b)	$w=\frac{22+4i}{6-8i}\times\frac{6+8i}{6+8i}$	M1
		$=\frac{100+200i}{100}=1+2i$	A1 A1 (3)
	(c)	$\arg z=\arctan\frac{4}{22}$	M1
		$=0.18$	A1 (2)
			(7 marks)
3.	(a)	$\sum_{r=1}^{n}(r-1)(r+2)=\sum_{r=1}^{n}r^2+\sum_{r=1}^{n}r-\sum_{r=1}^{n}2$	M1
		$=\frac{1}{6}n(n+1)(2n+1)+\frac{1}{2}n(n+1)-2n$	A1 A1
		[A1 for $-2n$, A1 for rest]	
		$=\frac{1}{6}n(2n^2+6n-8)$	
		Use factor n and use common denominator. (e.g.3, 6, 12)	M1
		$=\frac{1}{3}n(n^2+3n-4)=\frac{1}{3}(n-1)n(n+4)$ (✱)	A1 cso (5)
		Attempt complete factorisation	

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(b)	Use S(20) – S(4) = $\frac{1}{3}\times 19\times 20\times 24-\frac{1}{3}\times 3\times 4\times 8$	M1
	= 3008	A1 (2)
		(7 marks)
4. (a)	f(0.5) = –0.75, f(0.6) = 0.36	B1 B1
	$\alpha = \frac{0.5\times 0.36+0.6\times 0.75}{0.36+0.75}$	M1
	= 0.568 (3sf)	A1 (4)
(b)	$f'(x) = 2x+\frac{3}{x^2}$	M1 A1
	f(0.55) = –0.1520… f ′(0.55) = 11.0173…	B1
	$\alpha = 0.55 - \frac{-0.1520...}{11.0173...}$	M1
	= 0.564 (3sf)	A1 (5)
		(9 marks)
5. (a)	(i) 2 – i	B1
	(ii) $(2-i)^2 + b(2+i) + c = 0$ o.e.	M1
	Imaginary parts $b = -4$	B1
	Real parts $c + 3 + 2b = 0$	M1
	$c = 5$	A1 (5)
(b)	$(2+i)^3 = 2 + 11i$	B1
	$\alpha = + m(3+4i) + n(2+i) - 5 = 0$	M1
	Real parts $3m + 2n = 3$, Imaginary parts $8m + 2n = -22$	M1
	$m = -5$, $n = 9$	A1 A1 (5)
		(10 marks)
6. (a)	$\mathbf{A}^{-1}\mathbf{A}\mathbf{B} = \mathbf{A}^{-1}\mathbf{C}$	M1
	$\therefore$ $\mathbf{B} = \mathbf{A}^{-1}\mathbf{C}$ (✱)	A1 cso (2)
(b)	det **C** = 2	B1 (1)
(c)	Area of T_2 = det $\mathbf{C}\times\frac{1}{2}\times 5\times 3$ = $2\times\frac{1}{2}\times 5\times 3$	M1 M1
	= 15	A1 (3)

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(c)	Alternative $\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 0 & -5 & -3 \\ 0 & 5 & -3 \end{pmatrix}$	M1
	Area = $\frac{1}{2}\sqrt{50}\sqrt{18} = 15$	M1 A1 (3)
(d)	det $\mathbf{A} = 1$ $\quad \mathbf{A}^{-1} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	M1 A1
	$\mathbf{B} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$	M1 A1 (4)
(e)	Enlargement centre (0, 0), scale factor $\sqrt{2}$	B1 B1 (2)
		(12 marks)
7. (a)	$y = \frac{4}{x}$ $\quad \frac{dy}{dx} = -\frac{4}{x^2}$ $\quad y =$ and attempt $\frac{dy}{dx}$	M1
	$\frac{dy}{dx} = -\frac{1}{t^2}$ $\quad$ substitute $x = 2t$	M1
	Gradient of normal = t^2	M1
	Equation of normal is $\quad y - \frac{2}{t} = t^2(x - 2t)$	M1
	$y = t^2x + \frac{2}{t} - 2t^3 \quad$ (✱)	A1 cso (5)
(b)	At A $\quad 2t = -4, \quad t = -2$	B1
	Equation of normal is $\quad y = 4x + 15$	M1
	At B $\quad 4x + 15 = \frac{4}{x}$	M1 A1
	$4x^2 + 15x - 4 = 0 \quad \Rightarrow \quad (4x - 1)(x + 4) = 0$	
	$\therefore$ At B $\quad x = \frac{1}{4}$	M1 A1
	$y = 16$	A1 (7)
		(12 marks)

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8. (a) (i)	$f(k+1) - f(k)$	
	$= k^3 + 3k^2 + 3k + 1 - 10k - 10 + 15 - (k^3 - 10k + 15)$	M1
	$= 3k^2 + 3k - 9$	A2, 1, 0 (3)
(b)	$f(1) = 6 = 3\times2$ $\quad\therefore$ true for $n = 1$	B1
	$f(k+1) - f(k) = 3k^2 + 3k - 9 = 3(k^2 + k - 3)$	M1 A1
	$\therefore$ true for $n = k + 1$ if true for $n = k$, $\therefore$ true for $n \in \mathbb{Z}^+$ by induction	A1 cso (4)
(ii)	When $n = 1$, LHS $= 1(2)^1 = 2$; RHS $= 2\{1 + 0\} = 2$	B1
	true for $n = 1$	
	$\sum_{1}^{k+1} r\,2^r = 2\{1 + (k-1)2^k\} + (k+1)2^{k+1}$	M1 A1
	$= 2 + k\,2^{k+1} - 2^{k+1} + k\,2^{k+1} + 2^{k+1}$	
	$= 2(1 + k\,2^{k+1})$	
	$= 2\{1 + [(k+1) - 1]2^{k+1}\}$	M1 A1
	$\therefore$ true for $n = k + 1$ if true for $n = k$, $\therefore$ true for $n \in \mathbb{Z}^+$ by induction	A1 cso (6)
		(12 marks)