

GCE Examinations
Advanced Subsidiary / Advanced Level
Pure Mathematics
Module P4

Paper H

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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P4 Paper H – Marking Guide

1. (a) $f(r+1) - f(r) = (r+1)! - r!$
 $= r![(r+1) - 1] = r \times r!$ M1 A1
- (b) $\sum_{r=1}^n (r \times r!) = \sum_{r=1}^n [(r+1)! - r!]$ M1
 $= [2! + 3! + \dots + n! + (n+1)!] - [1! + 2! + 3! + \dots + n!]$ M1 A1
 $= (n+1)! - 1! = (n+1)! - 1$ A1 **(6)**
-
2. (a) $y(x^2 + 9) = 2x \therefore yx^2 - 2x + 9y = 0$ M1
 $x^2 - \frac{2}{y}x + 9 = 0 \therefore (x - \frac{1}{y})^2 - \frac{1}{y^2} + 9 = 0$ M1 A1
giving $x = \frac{1}{y} \pm \sqrt{\frac{1}{y^2} - 9}$ M1 A1
- (b) for x to be real, $\frac{1}{y^2} - 9 \geq 0$ M1
 $y^2 \geq 0 \therefore 1 \geq 9y^2$ so $y^2 \leq \frac{1}{9}$ A1
 $\therefore -\frac{1}{3} \leq y \leq \frac{1}{3}$ as required with $a = 3$ A1 **(8)**
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3. $x \frac{dy}{dx} + y(x+1) = 1; \frac{dy}{dx} + (\frac{x+1}{x})y = \frac{1}{x}$ M1 A1
int. fac. = $e^{\int \frac{1+x}{x} dx} = e^{x+\ln x} = e^x e^{\ln x} = xe^x$ M1 A2
 $\therefore xe^x \frac{dy}{dx} + y(x+1)e^x = e^x$ M1
 $\frac{d}{dx}(yxe^x) = e^x$ A1
 $xye^x = \int e^x dx = e^x + c$ M1
 $\therefore y = \frac{1}{x} + \frac{c}{xe^x}$ A1 **(9)**
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4. (a) let $f(x) = x^2 - \frac{3}{3x-2}$
 $f(1) = -2, f(2) = 3.25, f(1.5) = 1.05, f(1.3) = 0.111, f(1.2) = -0.435$ M1 A1
 f cont. over interval, change of sign \therefore root M1
 $\therefore 1.2 < \alpha < 1.3$ so $N = 12$ A1
- (b) $f(1.25) = -0.152, \therefore 1.25 < \alpha < 1.3$ M1 A1
 $f(1.275) = -0.0182 \therefore 1.275 < \alpha < 1.3$ A1
 $f(1.2875) = 0.0469 \therefore 1.275 < \alpha < 1.2875$
 $f(1.28125) = 0.0145 \therefore 1.275 < \alpha < 1.28125 \therefore \alpha = 1.28$ (2dp) M1 A1 **(9)**
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5. (a) $f(i) = 0 \therefore i^4 - 4i^3 + ki^2 - 4i + 13 = 0$ M1
 $1 + 4i - k - 4i + 13 = 0$ M1
 $\therefore 14 - k = 0$ so $k = 14$ A1
- (b) i is a root $\therefore -i$ is a root B1
 $\therefore (z-i)(z+i) = (z^2 + 1)$ is a factor M1 A1
 $\therefore f(z) = (z^2 + 1)(z^2 - 4z + 13)$ M1 A1
 $f(z) = 0 \Rightarrow z = \pm i$ or $z = \frac{4 \pm \sqrt{16-52}}{2}$ M1
 $\therefore z = \pm i$ or $2 \pm 3i$ A1 **(10)**

6.	(a)	require $\frac{d(r \sin \theta)}{d\theta} = 0$	M1	
		$r \sin \theta = a \sin \theta \sqrt{\sin 2\theta}$	A1	
		$\therefore \frac{d(r \sin \theta)}{d\theta} = a[\sin \theta \frac{1}{2}(\sin 2\theta)^{-\frac{1}{2}}(2\cos 2\theta) + \sqrt{\sin 2\theta}(\cos \theta)]$	M1 A1	
		giving $(\sin 2\theta)^{-\frac{1}{2}}[\sin \theta \cos 2\theta + \sin 2\theta \cos \theta] = 0$	M1	
		$(\sin 2\theta)^{-\frac{1}{2}}[\sin 3\theta] = 0$		
		$\therefore \sin 3\theta = 0, 3\theta = 0, \pi, 2\pi \dots$	A1	
		at A, $0 < \theta < \frac{\pi}{2} \therefore \theta = \frac{\pi}{3}$	A1	
	(b)	at A, $r^2 = a^2 \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2} a^2$	M1	
		side length = $OP = OA \sin \frac{\pi}{3} = \sqrt{\frac{\sqrt{3}}{2}} a \times \frac{\sqrt{3}}{2}$	M1	
		area = $OP^2 = \frac{\sqrt{3}}{2} a^2 \times \frac{3}{4} = \frac{3\sqrt{3}}{8} a^2$	A1	
	(c)	area used = $\frac{1}{2} \int_0^{\frac{\pi}{2}} a^2 \sin 2\theta \, d\theta$	M1	
		$= \frac{1}{2} a^2 [-\frac{1}{2} \cos 2\theta]_0^{\frac{\pi}{2}}$	A1	
		$= -\frac{1}{4} a^2 [\cos \pi - \cos 0] = \frac{1}{2} a^2$	A1	
		\therefore area not used = $\frac{3\sqrt{3}}{8} a^2 - \frac{1}{2} a^2 = \frac{1}{8} a^2 (3\sqrt{3} - 4)$	M1 A1	(15)

7.	(a)	$x = ke^{-t}, \frac{dx}{dt} = -ke^{-t}, \frac{d^2x}{dt^2} = ke^{-t}$	M1	
		$\therefore ke^{-t} + 5(-ke^{-t}) + 6ke^{-t} = 8e^{-t}$	M1	
		$2k = 8 \therefore k = 4$	A1	
	(b)	aux. eqn. $m^2 + 5m + 6 = 0$	M1	
		$(m + 3)(m + 2) = 0; m = -2, -3$	A1	
		gen. soln. $x = Ae^{-2t} + Be^{-3t} + 4e^{-t}$	A1	
		$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - 4e^{-t}$	M1	
		$t = 0, x = 1 \therefore 1 = A + B + 4$		
		$t = 0, \frac{dx}{dt} = 3 \therefore 3 = -2A - 3B - 4$	A1	
		solve simul. giving $A = -2, B = -1$	M1 A1	
		$\therefore x = 4e^{-t} - 2e^{-2t} - e^{-3t}$	A1	
	(c)	$\frac{dx}{dt} = -4e^{-t} + 4e^{-2t} + 3e^{-3t}$	M1	
		for S.P. = 0 $\therefore e^{-t}(3e^{-2t} + 4e^{-t} - 4) = 0$	M1	
		$e^{-t}(3e^{-t} - 2)(e^{-t} + 2) = 0$	M1	
		$\therefore e^{-t} = 0$ (no solns) or -2 (no solns) or $\frac{2}{3}$	A1	
		when $e^{-t} = \frac{2}{3}, x = 4(\frac{2}{3}) - 2(\frac{2}{3})^2 - (\frac{2}{3})^3$	M1	
		$= \frac{8}{3} - \frac{8}{9} - \frac{8}{27} = \frac{40}{27}$	A1	
		max occurs at $T = -\ln \frac{2}{3} = \ln \frac{3}{2}$	A1	(18)

Total (75)

Performance Record – P4 Paper H

Question no.	1	2	3	4	5	6	7	Total
Topic(s)	series	inequality	1 st order diff. eqn	numerical methods	complex nos.	polar coords	2 nd order diff. eqn	
Marks	6	8	9	9	10	15	18	75
Student								