# Mark Scheme (Pre-Standardisation) J anuary 2008 

## GCE

GCE Mathematics (6674/ 01)

## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

J anuary 2008

## 6674 Further Pure Mathematics FP1

Mark Scheme

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 1 | $\begin{aligned} & \text { Integrating factor }=e^{-3 x} \\ & \begin{aligned} \therefore \frac{d}{d x}\left(y e^{-3 x}\right)=x e^{-3 x} \end{aligned} \\ & \begin{aligned} \therefore\left(y e^{-3 x}\right)=\int x e^{-3 x} d x & =-\frac{x}{3} e^{-3 x}+\int \frac{1}{3} e^{-3 x} d x \\ & =-\frac{x}{3} e^{-3 x}-\frac{1}{9} e^{-3 x}(+c) \\ \therefore y & =-\frac{x}{3}-\frac{1}{9}+c e^{3 x} \end{aligned} \end{aligned}$ | B1 <br> M1 <br> M1 <br> A1 <br> A1ft <br> [5] |


| 2. | Use $(2 x+1)$ as factor to give $f(x)=(2 x+1)\left(x^{2}-6 x+10\right)$ Attempt to solve quadratic to give $x=\frac{6 \pm \sqrt{(36-40)}}{2}$ <br> Two complex roots are $=3 \pm \mathrm{i}$ | M1 A1 <br> M1 A1 <br> M1 A1 <br> (6) <br> [ 6 ] |
| :---: | :---: | :---: |
| 3. <br> (a) <br> (b) | Consider $\frac{(x+3)(x+9)-(3 x-5)(x-1)}{(x-1)}$, obtaining $\frac{-2 x^{2}+20 x+22}{(x-1)}$ <br> Factorise to obtain $\frac{-2(x-11)(x+1)}{(x-1)}$. <br> Identify $x=1, x=a$ and $x=b$ as significant <br> Check inequality ranges by suitable method <br> To obtain $x<-1, \quad 1<x<11$ | M1 A1 <br> M1 A1 <br> (4) <br> B1 <br> M1 <br> A1, A1 <br> (4) <br> [ 8 ] |
| 4. (a) <br> (b) | $f(0.7)=-0.195028497 \text { and } f(0.8)=0.297206781$ <br> Use $\frac{0.8-\alpha}{\alpha-0.7}=\frac{f(0.8)}{-f(0.7)}$ to obtain $\alpha=\frac{-0.8 f(0.7)+0.7 f(0.8)}{f(0.8)-f(0.7)}$ $=.739620991 \quad \text { Answer required to } 3 \mathrm{dp}$ $f^{\prime}(x)=6 x+1-\frac{1}{2} \sec ^{2}\left(\frac{x}{2}\right)$ <br> Use $x_{2}=0.75-\frac{f(0.75)}{f^{\prime}(0.75)}=0.741087218 \quad$ Answer required to 3 dp | B1, B1 <br> M1 <br> A1 <br> (4) <br> M1 A1 <br> M1 A1 <br> (4) <br> [ 8 ] |
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| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 5. (a) <br> (b) | Method to obtain partial fractions e.g. $5 r+4=A(r+1)(r+2)+\operatorname{Br}(r+2)+C r(r+1)$ And equating coefficients, or substituting values for $x$. $A=2, B=1, C=-3 \text { or } \frac{2}{r}+\frac{1}{r+1}-\frac{3}{r+2}$ $\begin{aligned} \sum_{r=1}^{n} \ldots & =\frac{2}{1}+\frac{1}{2}-\frac{3}{3} \\ & +\frac{2}{2}+\frac{1}{3}-\frac{3}{4} \\ & +\frac{2}{3}+\frac{1}{4}-\frac{3}{5} \quad=\quad 2+\frac{3}{2},-\frac{2}{n+1}-\frac{3}{n+2} \\ & +\ldots \\ & +\frac{2}{n-1}+\frac{1}{n}-\frac{3}{n+1} \\ & +\frac{2}{n}+\frac{1}{n+1}-\frac{3}{n+2} \\ & =\frac{7(n+1)(n+2)-4(n+2)-6(n+1)}{2(n+1)(n+2)}=\frac{7 n^{2}+11 n}{2(n+1)(n+2)} * \end{aligned}$ | M1 <br> A1 A1 A1 <br> (4) <br> M1 A1, A1 <br> M1 A1 <br> (5) |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| $6(a)$ <br> (b) <br> (c) <br> (d) | (i) Multiply top and bottom by conjugate to give $\frac{-2-i}{5}$ <br> ( ii ) Expand and simplify to give 3-4i $z^{2}-z=5-5 i, \quad\left\|z^{2}-z\right\|=5 \sqrt{2}$ $\arg \left(z^{2}-z\right)=-\frac{\pi}{4}$  <br> one mark for each point | M1 A1 <br> M1 A1 <br> (4) <br> M1A1 <br> (2) <br> M1 A1 <br> (2) <br> B1, <br> B1 <br> (2) <br> [10] |
| $\begin{array}{ll}7 & \text { (a) } \\ \\ \\ \\ \\ & \\ \text { (b) }\end{array}$ | Solve auxiliary equation $3 m^{2}-m-2=0$ to obtain $m=-\frac{2}{3}$ or 1 C.F is $A e^{-\frac{2}{3} x}+B e^{x}$ <br> Let $\mathrm{PI}=\lambda x^{2}+\mu x+v$. Find $y^{\prime}=2 \lambda x+\mu$, and $y^{\prime \prime}=2 \lambda$ and substitute into d.e. Giving $\lambda=-\frac{1}{2}, \mu=\frac{1}{2}$ and $v=-\frac{7}{4}$ $\therefore y=-\frac{1}{2} x^{2}+\frac{1}{2} x-\frac{7}{4}+A e^{-\frac{2}{3} x}+B e^{x}$ <br> Use boundary conditions: $\begin{aligned} & 2=-\frac{7}{4}+A+B \\ & y^{\prime}=-x+\frac{1}{2}-\frac{2}{3} A e^{-\frac{2}{3} x}+B e^{x} \text { and } 3=\frac{1}{2}-\frac{2}{3} A+B \end{aligned}$ <br> Solve to give $A=3 / 4, \quad B=3\left(\therefore y=-\frac{1}{2} x^{2}+\frac{1}{2} x-\frac{7}{4}+\frac{3}{4} e^{-\frac{2}{3} x}+3 e^{x}\right)$ | M1 A1 <br> A1 <br> M1 <br> A1 A1A1 <br> A1ft <br> (8) <br> M1A1 <br> M1 A1 <br> M1 A1 <br> (6) <br> [14] |


| Question <br> Number | Scheme | Marks |
| :---: | :---: | :---: |
| (b) <br> (c) | $a(3+2 \cos \theta)=4 a$ <br> Solve to obtain $\cos \theta=\frac{1}{2}$ <br> $\therefore \theta= \pm \frac{\pi}{3}$ and points are ( $4 a, \frac{\pi}{3}$ ) and ( $4 a,-\frac{5 \pi}{3}$ ) <br> Use area $=\frac{1}{2} \int r^{2} d \theta$ to give $\frac{1}{2} a^{2} \int(3+2 \cos \theta)^{2} d \theta$ <br> Obtain $\int(9+12 \cos \theta+2 \cos 2 \theta+2) d \theta$ <br> Integrate to give $11 \theta+12 \sin \theta+\sin 2 \theta$ <br> Use appropriate limits (e.g. $\frac{\pi}{3}$ and $\pi$, then double) <br> Find a third area of circle $=\frac{16 \pi a^{2}}{3}$ <br> Obtain required area $=\frac{38 \pi a^{2}}{3}-\frac{13 \sqrt{3} a^{2}}{2}$ <br> correct shape and position 5a and 4a marked 2a marked and passes through O | M1 <br> M1 <br> A1, A1 <br> (4) <br> M1 <br> A1 <br> M1 A1 <br> M1 <br> B1 <br> A1 A1 <br> (8) <br> B1 <br> B1 <br> B1 <br> (3) |

