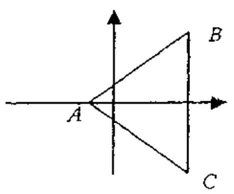
June 2005
6674 Pure Maths P4
Mark Scheme

Question Number	Scheme	Marks
1 (a)	$\frac{2}{4r^2-1} = \frac{1}{2r-1} - \frac{1}{2r+1}$	B1
	$\sum_{r=1}^{n} \frac{2}{4r^2-1} = \frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \cdots \frac{1}{2n-1} - \frac{1}{2n+1}$ Attempt Method of Differences	M1
	$= 1 - \frac{1}{2n+1}$ (*)	A1 c.s.o. (3)
(b)	Sum $= \left(\frac{1}{2}\right)\left[f(20) - f(10) \right]$ Use of (a) and $\sum_{1}^{20} - \sum_{1}^{10}$	M1
	$= \frac{1}{2}\left[1 - \frac{1}{41} - 1 + \frac{1}{21}\right] = \frac{10}{21 \times 41}$ or $\frac{10}{861}$	A1 c.a.o (2) 5
2.	(a) 1 − 3i is a root	B1
	$(z-1-3i)(z-1+3i)(z+\alpha) = (z^2-2z+10)(z+\alpha)$	
	$= z^3 + 6z + 20$	
	$10\alpha = 20 \Rightarrow \alpha = 2 \Rightarrow -2$ is a root M1 any complete method	M1 A1 3
	(b)  Position of points only in correct quadrants and negative x-axis	B1ft 1
	(c) $m_{AB} = \frac{3}{3} = 1, \quad m_{AC} = -1$ Full Method	M1
	$m_{AB} \cdot m_{AC} = -1 \Rightarrow$ triangle is right-angled	A1 2 6
	Alternatives	
	$AB^2 + AC^2 = 18 + 18 = 36 = BC^2$	
	Result follows by (converse of) Pythagoras, or any complete method.	M1 A1

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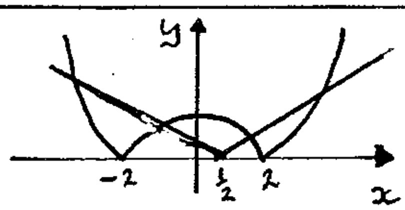
3.	$\frac{dy}{dx} + \frac{2}{1+x}y = \frac{1}{x(x+1)}$	Attempt $y' + Py = Q$ form	M1
	I.F. $= e^{\int \frac{2}{1+x}dx} = e^{2\ln(1+x)}, = (1+x)^2$		M1, A1
	$\therefore \quad y(1+x)^2 = \int \left(\frac{x+1}{x}\right)dx$ OR $\frac{d}{dx}\left(y(1+x)^2\right) = \frac{x+1}{x}$		M1 ($\int$ I.F.)
	i.e. $\left(y(1+x)^2 =\right) x + \ln x + (c)$		M1 A1
	$y = \frac{x + \ln x + c}{(1+x)^2}$		A1 c.a.o. (7)
4.	NB. $f(1) = 1.0\ldots$, $f(1.1) = 0.42\ldots$, $f(1.2) = -0.2937\ldots$ $f(1.15) = 0.078\ldots$, $f(1.4) = -2.05$.		
(a)	$f(1.2) = -0.2937\ldots$	f(1.2) to 1SF or better	B1
	$f(1.1) = 0.42\ldots$ and $f(1.15) = 0.078\ldots$	Attempt f(1.1), f(1.15)	M1
	$\therefore \quad \alpha = 1.2$		A1 c.a.o. (3)
(b)	$f'(x) = 6\cos 2x - e^x$		M1 A1
	$x_2 = 1.2 - \frac{-0.2937\ldots}{f'(1.2)}$		M1
	$= 1.16(2\ldots)$	A.W.R.T. 1.16	A1 (4)
(c)	$f(1.155) = 0.04\ldots\ldots$ $f(1.165) = -0.029\ldots$, $\Big\}$ (change of sign) $\therefore \alpha = 1.16$		M1, A1 (2)
	N.B. $f'(1.2) = -7.744\ldots$		(9)

5 (a).	$\frac{\pi}{2} + \arctan\frac{4}{6}$ or $\pi - \arctan\frac{6}{4}$ or equiv. in degrees		M1
	$\arg z = \underline{2.159}$		A1 c.a.o. (2)
(b)	$\lvert w\rvert = \sqrt{20} \Rightarrow \sqrt{20} = \frac{A}{\sqrt{5}}$	Full method for A using $\lvert w\rvert = \sqrt{20}$	M1
	$\Rightarrow \underline{A = 10}$		A1
	$w = \frac{A}{2-i} \times \left(\frac{2+i}{2+i}\right)$, $\underline{w = 4+2i}$		M1, A1 (4)
(c)	$\arg\left(\frac{w}{z}\right) = \arg w - \arg z = \arctan\left(\frac{2}{4}\right) - (a)$		M1
	$= 0.463... - 2.159$	2dp per letter	A1 √ (a)
	$= -1.695...$	awrt $\underline{-1.70}$	A1 (3)
ALT (c)	$\frac{w}{z} = -0.0769.... -0.6153....i$	Attempt $w \div z$ and use arctan	M1 ⑨
	$\Rightarrow \arg\left(\frac{w}{z}\right) = -\left[\pi - \arctan\left(\frac{0.6153...}{0.0769...}\right)\right]$	Expression (2dp)	A1
	$=$ awrt $\underline{-1.70}$		A1 (3)
6 (a)		 Shape. Symmetric about y-axis	B1
		 Shape. Vertex on positive x-axis	B1
		$-2,\ 2$	B1
		$\frac{1}{2}$	B1 (4)
(b)	$x^2 - 4 = 2x - 1$		M1
	$x^2 - 2x - 3 = 0 \Rightarrow x = \underline{3, -1}$		A1
	$x^2 - 4 = -(2x-1)$		M1
	$x^2 + 2x - 5 = 0, \Rightarrow x = \frac{-2 \pm \sqrt{4+20}}{2}$	Correct 3 term Quadratic = 0	A1,
	$x = \underline{\underline{-1 \pm \sqrt{6}}}$		A1 (5)
(c)	$x < -1-\sqrt{6}$; $-1 < x < \sqrt{6}-1$; $x > 3$ (√ surds)	Accept 3 s.f.	B1√; B1√; B1 (3) ⑫

7 (a)	$2m^2+5m+2=0$	Attempt aux eqn $\rightarrow m=$	M1
	$\Rightarrow \quad m=-\frac{1}{2}, -2$		
	$\therefore \quad x_{CF} = Ae^{-2t}+Be^{-\frac{1}{2}t}$	C.F.	A1
	Particular Integral: $x = pt+q$	P.I.	B1
	$\dot{x}=p$, $\ddot{x}=0$ and sub.		M1
	$\Rightarrow \quad 5p+2q+2pt = 2t+9 \rightarrow \underline{p=1, q=2}$		A1
	General solution $\quad x = \underline{\underline{Ae^{-2t}+Be^{-\frac{1}{2}t}+t+2}}$		A1$\int$ ($\int$ ms, p, q) (6)
(b)	$x=3, t=0 \Rightarrow 3 = A+B+2 \quad$ (or $A+B=1$)		M1
	$\dot{x} = -2Ae^{-2t} - \frac{1}{2}Be^{-\frac{1}{2}t}+1$	Attempt $\dot{x}$	M1
	$\dot{x}=-1, t=0 \Rightarrow -1 = -2A - \frac{1}{2}B + 1 \quad$ (or $4A+B=4$)	2 correct eqns	A1
	Solving $\rightarrow A=1, B=0$ and $\underline{x = e^{-2t}+t+2}$		A1 (4)
(c)	$\dot{x} = -2e^{-2t}+1 = 0$	$\dot{x}=0$	M1
	$\Rightarrow \quad t = \frac{1}{2}\ln 2$		A1
	$\ddot{x} = 4e^{-2t} > 0\ (\forall t) \therefore$ min		M1
	Min $\quad x = e^{-\ln 2} + \frac{1}{2}\ln 2 + 2$		
	$= \frac{1}{2} + \frac{1}{2}\ln 2 + 2$		
	$= \underline{\underline{\frac{1}{2}(5+\ln 2)}}$ ⊛		A1 c.s.o. (4)
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June 2005

Advanced Subsidiary/Advanced Level

General Certificate of Education

Subject **6674 Pure Mathematics** Paper **P4**

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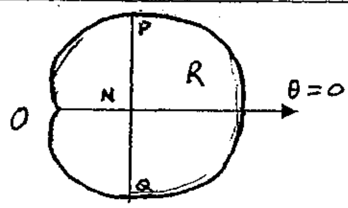
8.	(a) 		
	$4a(1+\cos\theta) = \dfrac{3a}{\cos\theta}$ or $r = 4a\left(1+\dfrac{3a}{r}\right)$		M1
	$4\cos^2\theta + 4\cos\theta - 3 = 0$ or $r^2 - 4ar - 12a^2 = 0$	dep	A1
	$(2\cos\theta - 1)(2\cos\theta + 3) = 0$ or $(r-6a)(r+2a) = 0$		M1
	$\cos\theta = \dfrac{1}{2}, \left(\theta = \dfrac{\pi}{3}\right)$ or $r = 6a$		A1
	Note $ON = 3a$		
	$PQ = 2 \times ON \tan\dfrac{\pi}{6} = 6\sqrt{3}a$ *	cso	M1 A1 6
	or $PQ = 2\times\sqrt{\left[(6a)^2 - (3a)^2\right]} = 2\sqrt{(27a^2)} = 6\sqrt{3}a$ *	cso	
	or any complete equivalent		
	(b) $2 \times \frac{1}{2}\int_0^{\pi/3} r^2 \, d\theta = \ldots\int 16a^2(1+\cos\theta)^2 \, d\theta$	$\int r^2 d\theta$	M1
	$= \ldots \int \left(1 + 2\cos\theta + \dfrac{1}{2} + \dfrac{1}{2}\cos 2\theta\right) d\theta$	$\cos^2\theta \to \cos 2\theta$	M1
	$= \ldots\left[\dfrac{3}{2}\theta + 2\sin\theta + \dfrac{1}{4}\sin 2\theta\right]$		A1
	$= 16a^2\left[\dfrac{\pi}{2} + \sqrt{3} + \dfrac{\sqrt{3}}{8}\right]$ $\left(= 2a^2\left[4\pi + 9\sqrt{3}\right] \approx 56.3a^2\right)$		use of their $\frac{\pi}{3}$ M1 A1
	Area of Δ POQ $= \frac{1}{2} 6\sqrt{3}a \times 3a$ or $9a^2\sqrt{3}$		B1
	$R = a^2(8\pi + 9\sqrt{3})$	cao	A1 7 (13)