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Edexcel GCE Final

Further Pure Maths
Unit no. 6674/01

June 2006

Mark Scheme
(Results)

Edexcel GCE

Further Pure Maths
6674/01

General Instructions

1. The total number of marks for the paper is 75.

2. Method (M) marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.

3. Accuracy (A) marks can only be awarded if the relevant method (M) marks have been earned.

4. (B) marks are independent of method marks.

5. Method marks should not be subdivided.

6. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. Indicate this action by 'MR' in the body of the script (but see also note 10).

7. If a candidate makes more than one attempt at any question:

   (a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.

   (b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

8. Marks for each question, or part of a question, must appear in the right-hand margin and, in addition, total marks for each question, even where zero, must be ringed and appear in the right-hand margin and on the grid on the front of the answer book. It is important that a check is made to ensure that the totals in the right-hand margin of the ringed marks and of the unringed marks are equal. The total mark for the paper must be put on the top right-hand corner of the front cover of the answer book.

9. For methods of solution not in the mark scheme, allocate the available M and A marks in as closely equivalent a way as possible, and indicate this by the letters 'OS' (outside scheme) put alongside in the body of the script.

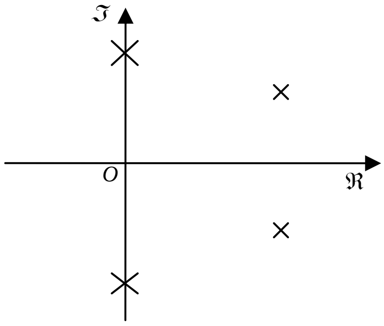
10. All A marks are 'correct answer only' (c.a.o.) unless shown, for example, as A1 f.t. to indicate that previous wrong working is to be followed through. In the body of the script the symbol $\sqrt{}$ should be used for correct f.t. and $\times$ for incorrect f.t. After a misread, however, the subsequent A marks affected are treated as A f.t., but manifestly absurd answers should never be awarded A marks.

11. Ignore wrong working or incorrect statements following a correct answer.

June 2006
6674 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $2z + \mathrm{i}w = -1$	
	$\mathrm{i}z - \mathrm{i}w = 3\mathrm{i} - 3$	
	Adding $2z + \mathrm{i}z = -4 + 3\mathrm{i}$ Eliminating either variable	M1
	$z = \dfrac{-4+3\mathrm{i}}{2+\mathrm{i}}$	A1
	$z = \dfrac{-4+3\mathrm{i}}{2+\mathrm{i}} \times \dfrac{2-\mathrm{i}}{2-\mathrm{i}}$	M1
	$= \dfrac{-8+3+4\mathrm{i}+6\mathrm{i}}{5}$	
	$= -1+2\mathrm{i}$	A1 (4)
	(b) $\arg z = \pi - \underline{\arctan 2}$	<u>M1</u>
	≈ 2.03 cao	A1 (2)
		[6]
2.	Use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$	B1
	Limits are $\frac{\pi}{8}$ and $\frac{\pi}{4}$	B1
	$16a^2\cos^2 2\theta = 8a^2(1+\cos 4\theta)$	M1
	$\int(1+\cos 4\theta)\,\mathrm{d}\theta = \theta + \dfrac{\sin 4\theta}{4}$	M1 A1
	$A = 4a^2\left[\theta + \dfrac{\sin 4\theta}{4}\right]_{\pi/8}^{\pi/4}$	
	$= a^2\left[4\left(\dfrac{\pi}{4} - \dfrac{\pi}{8}\right) + (0-1)\right]$	M1
	$= a^2\left(\dfrac{\pi}{2} - 1\right) = \dfrac{1}{2}a^2(\pi - 2)$ * cso	A1 (7)
		[7]

June 2006
6674 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Marks
3.	(a) $y' = 3\sin 2x + 6x\cos 2x$	M1
	$y'' = 12\cos 2x - 12x\sin 2x$	A1
	Substituting $12\cos 2x - 12x\sin 2x + 12x\sin 2x = k\cos 2x$	M1
	$k = 12$	A1 (4)
	(b) General solution is $y = A\cos 2x + B\sin 2x + 3x\sin 2x$	B1
	$(0, 2) \Rightarrow A = 2$	B1
	$\left(\frac{\pi}{4}, \frac{\pi}{2}\right) \Rightarrow \frac{\pi}{2} = B + \frac{3\pi}{4} \Rightarrow B = -\frac{\pi}{4}$	M1
	$y = 2\cos 2x - \frac{\pi}{4}\sin 2x + 3x\sin 2x$ Needs $y = \ldots$	A1 (4)
		[8]
4.	(a) 3 + 2i is a solution	B1
	$(x-3-2\mathrm{i})(x-3+2\mathrm{i}) = x^2 - 6x + 13$	M1
	$\mathrm{f}(x) = (x^2 - 6x + 13)(x^2 + ax + b)$	
	$b = 6$	B1
	Coefficients of x^3 $a - 6 = -6$ or equivalent	M1
	$a = 0$	A1
	$x^2 + 6 = 0 \Rightarrow x = \sqrt{6}\mathrm{i}, -\sqrt{6}\mathrm{i}$	M1A1 (7)
	(b) 	
	Conjugate complex pair on imaginary axis	B1
	Conjugate complex pair in correct quadrants	B1 (2)
		[9]

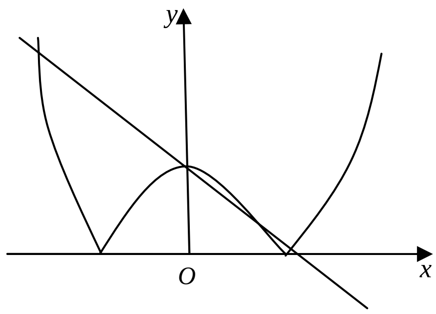
June 2006
6674 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Marks
5.	(a) $(2r+1)^3 = 8r^3+12r^2+6r+1$	
	$(2r-1)^3 = 8r^3-12r^2+6r-1$	
	$(2r+1)^3-(2r-1)^3 = 24r^2+2 \qquad (A=24,\ B=2)$	M1 A1 (2)
	Accept $r=0 \Rightarrow B=2$ and $r=1 \Rightarrow A+B=26 \Rightarrow A=24$	
	M1 for both	
	(b) $\cancel{3^3} - 1^3 = 24\times 1^2+2$	
	$\cancel{5^3} - \cancel{3^3} = 24\times 2^2+2$	
	M	
	$(2n+1)^3 - \cancel{(2n-1)^3} = 24\times n^2+2$	
	$(2n+1)^3 - 1^3 = 24\sum_{r=1}^{n} r^2 + \underline{2n}$ <u>ft</u> their B	M1 A1 <u>A1ft</u>
	$\sum_{r=1}^{n} r^2 = \dfrac{8n^3+12n^2+4n}{24}$	M1
	$= \dfrac{1}{6}n\left(2n^2+3n+1\right) = \dfrac{1}{6}n(n+1)(2n+1)$ * cso	A1 (5)
	(c) $\sum_{r=1}^{40}(3r-1)^2 = \sum_{r=1}^{40}(9r^2-6r+1)$	M1
	$= 9\times\dfrac{1}{6}\times 40\times 41\times 81 - 6\times\dfrac{1}{2}\times 40\times 41 + 40$	M1
	$= 194380$	A1 (3)
		[10]

June 2006
6674 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Marks
6.	(a) $\mathrm{f}(0.24) \approx -0.058,\ \mathrm{f}(0.28) = 0.089$ accept 1sf	M1
	Change of sign (and continuity) $\Rightarrow\ \alpha \in (0.24, 0.28)$	A1 (2)
	(b) $\mathrm{f}(0.26) \approx 0.017\ \ (\Rightarrow\ \ \alpha \in (0.24, 0.26))$ accept 1sf	M1
	$\mathrm{f}(0.25) \approx -0.020\ \ (\Rightarrow\ \ \alpha \in (0.25, 0.26))$	
	$\mathrm{f}(0.255) \approx -0.001\ \ \Rightarrow\ \ \alpha \in (0.255, 0.26)$	M1 A1 (3)
	(c) $\mathrm{f}(11) \approx 0.0534$ at least 3sf	B1
	$\mathrm{f}'(x) = \frac{2\cos\sqrt{x}}{\sqrt{x}} + \frac{1}{4}$	M1 A1
	$\mathrm{f}'(11) \approx -0.3438$ at least 2sf	A1
	$\beta \approx 11 + \frac{0.0534}{0.3438} \approx 11.16$ cao	M1 A1 (6)
		[11]
	If $\mathrm{f}'(11) \approx -0.3438$ is produced without working, this is to be accepted for three marks M1 A1 A1.	

June 2006
6674 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Marks
7.	(a) $2x^2 + x - 6 = 6 - 3x$	M1
	Leading to $x^2 + 2x - 6 = 0$	
	$(x+1)^2 = 7 \Rightarrow x = -1 \pm \sqrt{7}$ surds required	M1 A1
	$-2x^2 - x + 6 = 6 - 3x$	M1
	Leading to $2x^2 - 2x = 0 \Rightarrow x = 0, 1$	A1, A1 (6)
	(b) Accept if parts (a) and (b) done in reverse order	
	 Curved shape	B1
	Line	B1
	At least 3 intersections	B1 (3)
	(c) Using all 4 CVs and getting all into inequalities	M1
	$x > \sqrt{7} - 1, \quad x < -\sqrt{7} - 1$ both	A1ft
	ft their greatest positive and their least negative CVs	
	$0 < x < 1$	A1 (3)
		[12]

June 2006
6674 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Marks
8.	(a) $\int \frac{2}{120-t}\,\mathrm{d}t = -2\ln(120-t)$	B1
	$\mathrm{e}^{-2\ln(120-t)} = (120-t)^{-2}$	M1 A1
	$\frac{1}{(120-t)^2}\frac{\mathrm{d}S}{\mathrm{d}t} + \frac{2S}{(120-t)^3} = \frac{1}{4(120-t)^2}$	
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{S}{(120-t)^2}\right) = \frac{1}{4(120-t)^2}$ or integral equivalent	M1
	$\frac{S}{(120-t)^2} = \frac{1}{4(120-t)}(+C)$	M1 A1
	$(0, 6) \Rightarrow 6 = 30 + 120^2 C \Rightarrow C = -\frac{1}{600}$	M1
	$S = \frac{120-t}{4} - \frac{(120-t)^2}{600}$ accept $C =$ awrt -0.0017	A1 (8)
	(b) $\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{1}{4} + \frac{2(120-t)}{600}$	M1
	$\frac{\mathrm{d}S}{\mathrm{d}t} = 0 \Rightarrow t = 45$	M1 A1
	Substituting $S = 9\frac{3}{8}$ (kg)	A1 (4)
		[12]

Question Number	Scheme	Marks
8.Contd.	Alternative forms for S are $$S = 6 + \frac{3t}{20} - \frac{t^2}{600} = \frac{(t+30)(120-t)}{600}$$ $$= \frac{3600 + 90t - t^2}{600} = \frac{5625 - (t-45)^2}{600}$$	
	Alternative for part (b)	
	S can be found without finding t	
	Using $\frac{\mathrm{d}S}{\mathrm{d}t} = 0$ in the original differential equation	
	$$\frac{2S}{120-t} = \frac{1}{4}$$	M1
	Substituting for t into the answer to part (a) $$S = 2S - \frac{64S^2}{600}$$	M1 A1
	Solving to $\quad S = 9\frac{3}{8}$ (kg)	A1 (4)