Paper Reference(s)

6674

Edexcel GCE

Pure Mathematics P4
(New Syllabus)
Advanced/Advanced Subsidiary

Monday 14 January 2002 − Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P4), the paper reference (6674), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. Given that $z = 22 + 4\text{i}$ and $\dfrac{z}{w} = 6 - 8\text{i}$, find

(*a*) w in the form $a + b\text{i}$, where a and b are real,

(3)

(*b*) the argument of z, in radians to 2 decimal places.

(2)

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2. Find the set of values for which

$$|x - 1| > 6x - 1.$$

(5)

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3. (*a*) Prove that $\sum_{r=1}^{n} (r + 1)(r - 1) = \frac{1}{6} n (n - 1)(2n + 5)$.

(5)

(*b*) Deduce that $n(n - 1)(2n + 5)$ is divisible by 6 for all $n > 1$.

(2)

4.

$$\text{f}(x) = x^3 + x - 3.$$

The equation $\text{f}(x) = 0$ has a root, α, between 1 and 2.

(*a*) By considering $\text{f}'(x)$, show that α is the only real root of the equation $\text{f}(x) = 0$.

(3)

(*b*) Taking 1.2 as your first approximation to α, apply the Newton-Raphson procedure once to $\text{f}(x)$ to obtain a second approximation to α. Give your answer to 3 significant figures.

(2)

(*c*) Prove that your answer to part (*b*) gives the value of α correct to 3 significant figures.

(2)

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5. (a) Given that $2 + \mathrm{i}$ is a root of the equation

$$z^2 + bz + c = 0, \text{ where } b \text{ and } c \text{ are real constants,}$$

(i) write down the other root of the equation,

(ii) find the value of b and the value of c.

(5)

(b) Given that $2 + \mathrm{i}$ is a root of the equation

$$z^3 + mz^2 + nz - 5 = 0, \text{ where } m \text{ and } n \text{ are real constants,}$$

find the value of m and the value of n.

(5)

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6. (a) Find the general solution of the differential equation

$$t\frac{\mathrm{d}v}{\mathrm{d}t} - v = t, \quad t > 0$$

and hence show that the solution can be written in the form $v = t(\ln t + c)$, where c is an arbitrary constant.

(6)

(b) This differential equation is used to model the motion of a particle which has speed v m s^{-1} at time t s. When $t = 2$ the speed of the particle is 3 m s^{-1}. Find, to 3 significant figures, the speed of the particle when $t = 4$.

(4)

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7. (a) Show that $y = \frac{1}{2}x^2\mathrm{e}^x$ is a solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + y = \mathrm{e}^x.$$

(4)

(b) Solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + y = \mathrm{e}^x.$$

given that at $x = 0$, $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$.

(9)

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8. The curve C has polar equation $r = 3a \cos \theta$, $-\frac{\pi}{2} \leq \frac{\pi}{2}$. The curve D has polar equation $r = a(1 + \cos \theta)$, $-\pi \leq \theta < \pi$. Given that a is a positive constant,

(*a*) sketch, on the same diagram, the graphs of C and D, indicating where each curve cuts the initial line.

(4)

The graphs of C intersect at the pole O and at the points P and Q.

(*b*) Find the polar coordinates of P and Q.

(3)

(*c*) Use integration to find the exact value of the area enclosed by the curve D and the lines $\theta = 0$ and $\theta = \frac{\pi}{3}$.

(7)

The region R contains all points which lie outside D and inside C.

Given that the value of the smaller area enclosed by the curve C and the line $\theta = \frac{\pi}{3}$ is

$$\frac{3a^2}{16}(2\pi - 3\sqrt{3}),$$

(*d*) show that the area of R is πa^2.

(4)

END