

2. (a) Using the formulae for $\sum_{r=1}^n r$, $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r^3$, show that

$$\sum_{r=1}^n r(r+1)(r+3) = \frac{1}{12}n(n+1)(n+2)(3n+k),$$

where k is a constant to be found.

(7)

(b) Hence evaluate $\sum_{r=21}^{40} r(r+1)(r+3)$.

(2)



4. Given that α is the only real root of the equation

$$x^3 - x^2 - 6 = 0$$

(a) show that $2.2 < \alpha < 2.3$ (2)

(b) Taking 2.2 as a first approximation to α , apply the Newton-Raphson procedure once to $f(x) = x^3 - x^2 - 6$ to obtain a second approximation to α , giving your answer to 3 decimal places. (5)

(c) Use linear interpolation once on the interval [2.2, 2.3] to find another approximation to α , giving your answer to 3 decimal places. (3)



5. $\mathbf{R} = \begin{pmatrix} a & 2 \\ a & b \end{pmatrix}$, where a and b are constants and $a > 0$.

(a) Find \mathbf{R}^2 in terms of a and b . (3)

Given that \mathbf{R}^2 represents an enlargement with centre $(0, 0)$ and scale factor 15,

(b) find the value of a and the value of b . (5)



6. The parabola C has equation $y^2 = 16x$.

(a) Verify that the point $P(4t^2, 8t)$ is a general point on C . (1)

(b) Write down the coordinates of the focus S of C . (1)

(c) Show that the normal to C at P has equation
$$y + tx = 8t + 4t^3$$
 (5)

The normal to C at P meets the x -axis at the point N .

(d) Find the area of triangle PSN in terms of t , giving your answer in its simplest form. (4)



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Question 6 continued

Lined writing area for the answer to Question 6.

Q6

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(Total 11 marks)



M 3 5 1 4 6 A 0 1 9 2 4

7.
$$\mathbf{A} = \begin{pmatrix} a & -2 \\ -1 & 4 \end{pmatrix}$$
, where a is a constant.

(a) Find the value of a for which the matrix \mathbf{A} is singular. (2)

$$\mathbf{B} = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$$

(b) Find \mathbf{B}^{-1} . (3)

The transformation represented by \mathbf{B} maps the point P onto the point Q .

Given that Q has coordinates $(k - 6, 3k + 12)$, where k is a constant,

(c) show that P lies on the line with equation $y = x + 3$. (3)



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Question 7 continued

Lined area for writing the answer to Question 7.

(Total 8 marks)

Q7



8. Prove by induction that, for $n \in \mathbb{Z}^+$,

(a) $f(n) = 5^n + 8n + 3$ is divisible by 4, (7)

(b) $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix}$ (7)



Question 8 continued

Handwriting practice lines for the answer to Question 8. The page contains 22 horizontal lines.

(Total 14 marks)

Q8

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TOTAL FOR PAPER: 75 MARKS

END

