GCE Examinations

Pure Mathematics Module P4

Advanced Subsidiary / Advanced Level

Paper A

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 8 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Solomon Press

Written by Shaun Armstrong & Chris Huffer

1. $$f(z) \equiv z^3 - 5z^2 + 17z - 13.$$

(a) Show that $(z - 1)$ is a factor of $f(z)$. **(1 mark)**

(b) Hence find all the roots of the equation $f(z) = 0$, giving your answers in the form $a + ib$ where a and b are integers.

(5 marks)

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2. Find the general solution of the differential equation

$$x\frac{dy}{dx} + 3y = \frac{e^x}{x^2},$$

giving your answer in the form $y = f(x)$. **(6 marks)**

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3. *(a)* Express $\frac{1}{r(r+1)}$ in partial fractions. **(2 marks)**

(b) Hence, or otherwise, find

$$\sum_{r=3}^{35} \frac{1}{r(r+1)},$$

giving your answer as a fraction in its lowest terms. **(4 marks)**

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4. Find the set of values of x for which

$$\frac{(x-3)^2}{x+1} < 2.$$ **(7 marks)**

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5. *(a)* Sketch the curve with polar equation $r = a\cos 3\theta$, $a > 0$, for $0 \leq \theta \leq \pi$. **(3 marks)**

(b) Show that the total area enclosed by the curve $r = a\cos 3\theta$ is $\frac{\pi a^2}{4}$. **(6 marks)**

6.

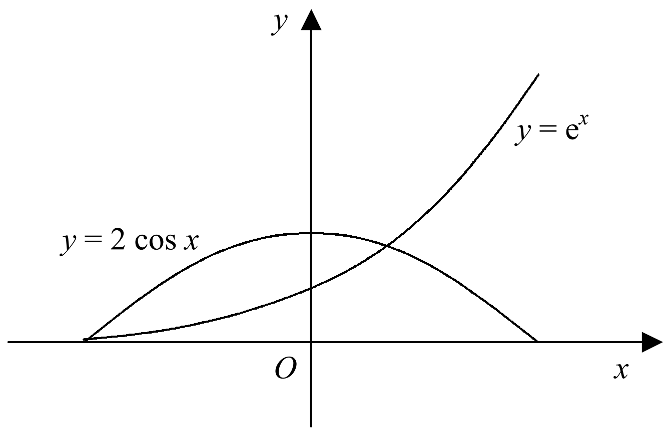

Fig. 1

Figure 1 shows the curves $y = 2\cos x$ and $y = e^x$ in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

Given that $f(x) \equiv e^x - 2\cos x$,

(a) write down the number of solutions of the equation $f(x) = 0$ in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

(1 mark)

(b) Show that the equation $f(x) = 0$ has a solution, α, in the interval $[0, 1]$. **(2 marks)**

(c) Using 0.5 as a first approximation to α, use the Newton-Raphson process once to find an improved estimate for α, giving your answer correct to 2 decimal places.

(4 marks)

(d) Show that the estimate of α obtained in part *(c)* is accurate to 2 decimal places.

(2 marks)

There is another root, β, of the equation $f(x) = 0$ in the interval $[-2, -1]$.

(e) Use linear interpolation once on this interval to estimate the value of β, giving your answer correct to 2 decimal places.

(3 marks)

Turn over

7. The complex numbers z and w are such that

$$z = \frac{A}{1-\mathrm{i}} \quad \text{and} \quad w = \frac{B}{2+\mathrm{i}},$$

where A and B are real.

Given that $z + w = 6$,

(a) find A and B. **(6 marks)**

z and w are represented by the points P and Q respectively on an Argand diagram.

(b) Show P and Q on the same Argand diagram. **(5 marks)**

(c) Find the distance PQ in the form $a\sqrt{5}$. **(3 marks)**

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8. *(a)* Find the values of p and q such that $x = p\cos t + q\sin t$ satisfies the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 3x = \sin t.$$ **(6 marks)**

(b) Hence find the solution of this differential equation for which $x = 1$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{2}$ at $t = 0$.

(9 marks)

END