Review Exercise 1

Exercise 1A

1. (a) $\left|z_1 z_2^{\,*}\right| = 5\sqrt{5}$, $\tan\arg(z_1 z_2^{\,*}) = -\frac{1}{2}$

(b) $\left|\frac{z_1}{z_2}\right| = \frac{\sqrt{5}}{5}$, $\tan\arg\left(\frac{z_1}{z_2}\right) = -\frac{1}{2}$

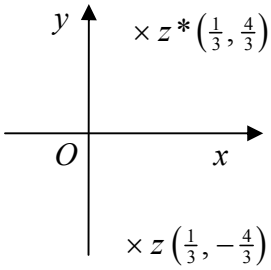
2. (a) $\frac{1}{2}$

(b) $-\frac{1}{4}$

3. (b) $\frac{3\pi}{4}$

4. (a) $\frac{1}{3} - \frac{4}{3}\mathrm{i}$

(b)



(c) $z = \frac{\sqrt{17}}{3}\cos(-76^\circ) + \mathrm{i}\frac{\sqrt{17}}{3}\sin(-76^\circ)$

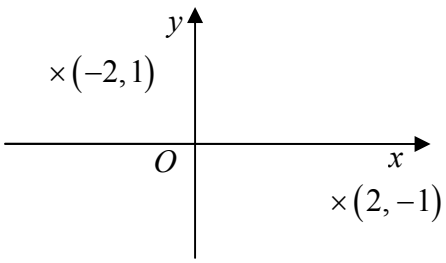
$z^* = \frac{\sqrt{17}}{3}\cos 76^\circ + \mathrm{i}\frac{\sqrt{17}}{3}\sin 76^\circ$

5. (a) (i) $\frac{2\pi}{3}$ **(ii)** $\frac{\pi}{6}$

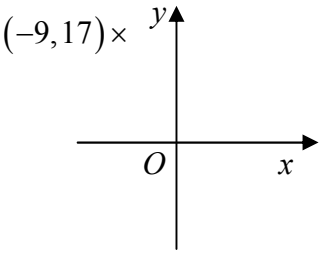
(b) $0 + \mathrm{i}$; $\frac{\pi}{2}$

6. (a) $2 - \mathrm{i}$ and $2 + \mathrm{i}$

(b)



7. (a)



(b) 2.06

(c) $1 - 2\mathrm{i}$

8. (a) $-1 + 2\mathrm{i}$

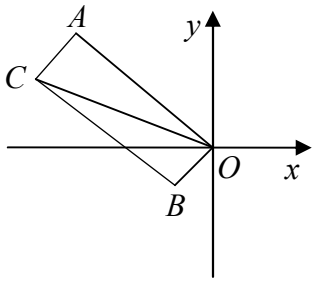
(b) 2.03

9. (b) $\frac{\sqrt{2}}{2}$; $-\frac{3\pi}{4}$

10. (a) $2\sqrt{2}$; $\frac{3\pi}{4}$

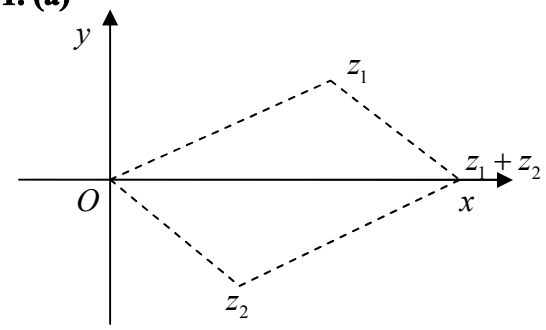
(b) $\frac{\sqrt{2}}{4}$; $-\frac{3\pi}{4}$

(c)



(d) 90°

11. (a)



(b) $\frac{1}{z_1} = \frac{\sqrt{3}}{4} - \frac{1}{4}\mathrm{i}$

$\frac{1}{z_2} = \frac{1}{2} + \frac{1}{2}\mathrm{i}$

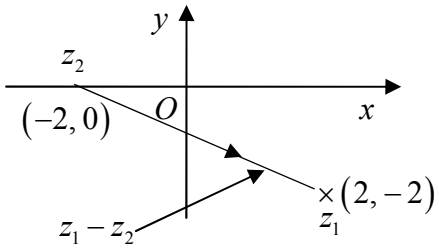
(c) $A = 4$, $B = 2$

12. (a) $A = -1, B = 5$

(b) 2

13. (b) $2 - 2\text{i}$ and -2

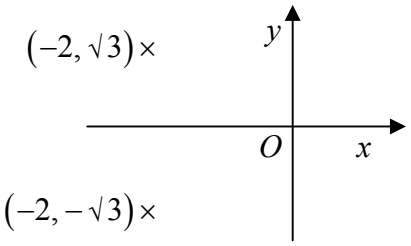
(c)



(e) $-\dfrac{\pi}{2}$

14. (a) $-2 + \text{i}\sqrt{3}$ and $-2 - \text{i}\sqrt{3}$

(b)



(c) (i) Both moduli = $\sqrt{7}$

(ii) arg $(-2 + \text{i}\sqrt{3}) = 2.43$;

arg $(-2 - \text{i}\sqrt{3}) = -2.43$

15. $1 + \text{i}\lambda$

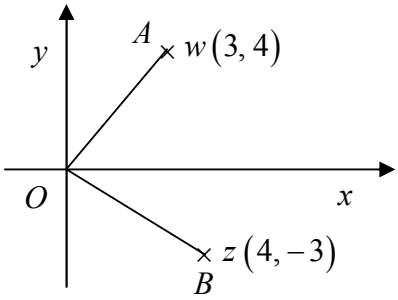
16. (a) $\sqrt{29}$

(b) -0.38

(c) 29

17. (a) $z = 4 - 3\text{i}$, $w = 3 + 4\text{i}$

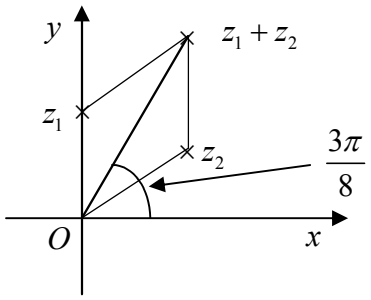
(b)



(c) 90°

18. (a) $|z_1| = 1$, arg $z_1 = \dfrac{\pi}{2}$, $|z_2| = 1$, arg $z_2 = \dfrac{\pi}{4}$

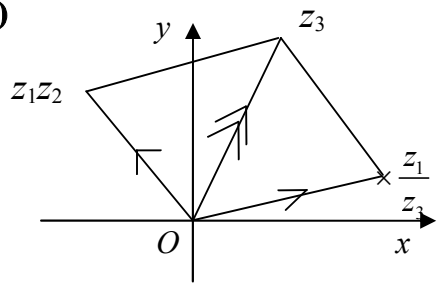
(b)



19. (a) (i) $-1 + 2\text{i}$

(ii) $\dfrac{11}{5} + \dfrac{2}{5}\text{i}$

(b)

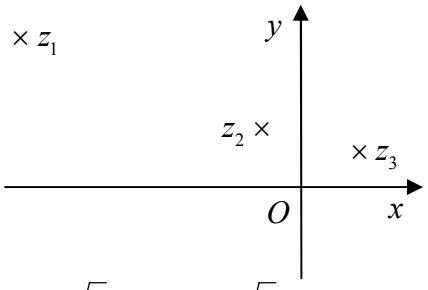


(c) $\dfrac{6}{5} + \dfrac{12}{5}\text{i}$

20. (a) 2.68

(b) $p = 6, q = 4$

(c)

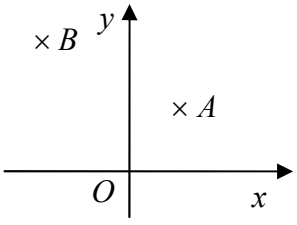


21. (a) $(2 - 2\sqrt{3}) + (2 + 2\sqrt{3})\text{i}$

(b) 1.83

(c) $4\sqrt{2}$

(d)



(e) $2\sqrt{5}$

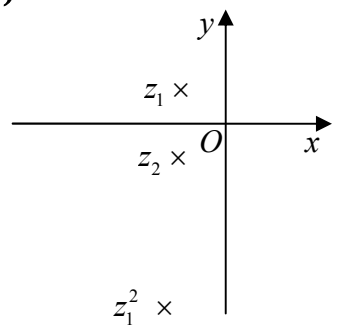
22. (a) $z_1 = -3 + 4\text{i},\ z_2 = -3 - 4\text{i}$

(b) $-7 - 24\text{i}$

(c) 25

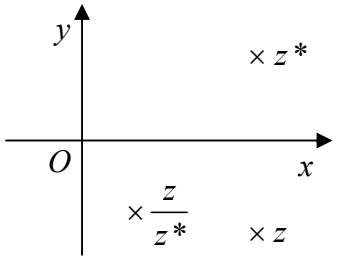
(d) -1.85

(e)



23. (b) 1

(d)



(e) $x^2 - 2\sqrt{3}\,x + 4 = 0$

24. (a) $\sqrt{2}\ ;\ \frac{\pi}{4}$

(b) $\sqrt{2}\ ;\ -\frac{\pi}{4}$

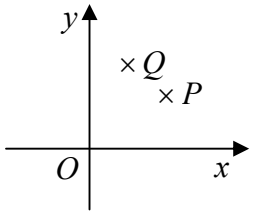
(c) $3 - 4\text{i}$

(d) 25

25. (b) 6

(c) 20

26. (a)



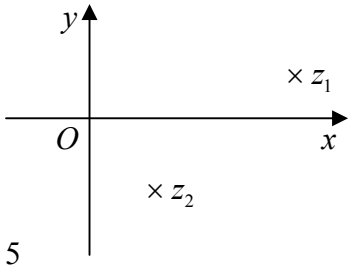
(b) $|z_1| = 2\sqrt{2}$, $|z_2| = \sqrt{10}$, $PQ = \sqrt{2}$

(d) $-1 + \text{i}$

27. (b) $p = 4,\ q = -2$

(c) $-\frac{\pi}{4}$

28. (a)



(b) 5

(c) $\frac{7}{13} + \frac{17}{13}\,\text{i}$

(d) 1.18

(e) $p = -7,\ q = -5$

29. (a) $z^2 = (a^2 - b^2) + 2ab\,\text{i},\ \frac{1}{z} = \frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2}\,\text{i}$

(c) $\tan(\arg z^2) = \frac{2ab}{a^2 - b^2}$, $\tan(\arg \frac{1}{z}) = -\frac{b}{a}$

30. 1.444

31. $1\frac{1}{3}$

32. 1.325

33. (a) $3x^2 - 12$

(b) 0.6015

34. (a) 1.89

35. [1.3, 1.35]

36. 1.18

37. (b) [1.875, 2]

38. (a) f(2) = 0.109, f(2.5) = −0.402

(b) 2.11

39. (b) 1.432

40. (b) 0.68

41. (a) [4.257, 4.2575]

(b) 4.257

42. (a) 0.401

(b) $4x - \frac{1}{x^2}$

(c) 0.361

43. (b) [0.255, 0.26]

44. (c) 1.729

45. (a) −1.25

(b) −0.33

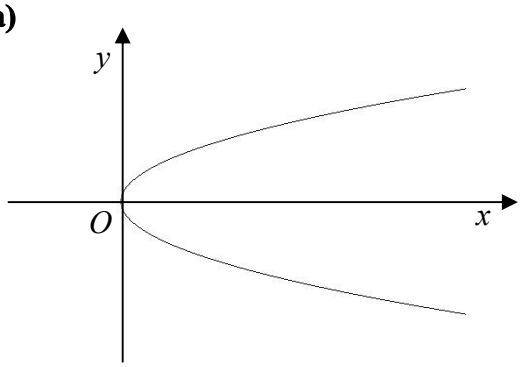
(c) [1.875, 1.9]

47. (a) (4, 0)

(b) $4x - 3y - 16 = 0$

(c) (1, −4)

48. (a)



(b) $60\sqrt{2}$

49. $4\sqrt{15}$

50. (a) 8

(b) $y = 2x + 4$

(c) 4

51. (a) $y = \frac{4}{5}x + \frac{8}{5}$

(b) $(-5, -2.4)$

52. (a) $x + 4y = 24$

(b) $6\sqrt{7}$

53. (a) $5x - 4y - 9 = 0$

(b) $(-3.2, -6.25)$

54. $(-8, 1)$

55. (a) $t = \frac{1}{2}$, $P\ (6, 24)$

(b) $y = 2x + 12$

(c) $y = -4x + 48$

56. (b) $(3\sqrt{6}, 4\sqrt{6})$ and $(-3\sqrt{6}, -4\sqrt{6})$

59. (b) $(-\frac{4}{3}, -12)$ and $(12, \frac{4}{3})$

61. (b) $(apq, a(p + q))$

(c) $p = 4 - q$

62. (b) $\left(a\left(\frac{t^2+2}{t}\right)^2, -2a\left(\frac{t^2+2}{t}\right)\right)$

63. (b) $\left(-\frac{c}{t^3}, -ct^3\right)$

64. (c) 1

65. (b) $(2cp, 0)$

(c) $\frac{c}{p}\left(1 + p^4\right)^{\frac{1}{2}}$

(d) $\left(\frac{c}{3}, 3c\right)$

66. (b) $q = -p - \frac{2}{p}$

(c) $\frac{2}{3}$

67. (a) (8, 0)

(b) $x = -8$